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Analysis of Thinking Mathematics Based on Van de Walle's: A Case Study in Students Grade 5th

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ABSTRACT

This study explores fifth-grade students' mathematical thinking processes, specifically how they interpret and solve addition and subtraction problems in story-based contexts. Using a qualitative case study approach, five students from an elementary school in Yogyakarta were examined through interviews, observations, and mathematics problem-solving tasks. The data were analyzed using inductive thematic coding to uncover patterns in computational strategy and conceptual understanding. Results showed that students employed diverse techniques such as horizontal and vertical stacking, semantic and computational sentence transformation, and even-number decomposition. These strategies reflected varying levels of comprehension regarding commutative and associative mathematical properties. Moreover, students who had difficulty translating story problems into mathematical expressions typically showed weak understanding of operational meaning and structure. The study found that most students shifted from semantic to computational forms for ease in calculation. This suggests the need for instructional support to enhance students' conceptual grasp of operations. Overall, this study provides insights into how mathematical thinking can be developed through contextual learning and supports integrating meaningful problem-solving practices into mathematics education.

Keywords:
Addition and subtraction, computational structure, elementary students, mathematical thinking, semantic structure

INTRODUCTION

Mathematics is a compulsory subject that plays a fundamental role in students' academic development. It is essential for understanding everyday phenomena and for supporting advances in science and technology (Gravemaster et al., 2017). Mathematics also fosters a range of higher-order thinking skills, including logical, analytical, systematic, critical, and creative thinking, as well as collaborative abilities. Despite its importance, mathematics is often perceived by students as difficult. This perception is reflected in low learning outcomes, which are further evidenced by the 2019 PISA results, where Indonesia ranked 73rd out of 79 participating countries in mathematics (Hewi & Shaleh, 2020). These conditions highlight the crucial role of teachers in designing meaningful learning experiences that align with students' real-life contexts.

The ability to perform addition and subtraction is a foundational mathematical skill that students must master in order to interpret and apply arithmetic operations effectively (Carpenter et al., 2015). Understanding the meaning of these operations is essential for developing conceptual understanding, connecting mathematics to everyday life, and solving more complex problems. However, a common challenge in mathematics instruction is the emphasis on algorithms and procedural rules without sufficient attention to the underlying meanings of operations. As a result, students often experience difficulties when applying addition and subtraction in real-world situations or in solving contextual problems.

According to Walle et al. (2016), addition and subtraction can be understood through general situations that emphasize underlying problem structures rather than surface features. Carpenter et al. (2015) classify addition and subtraction problems into three structural categories: join and separate, part–whole, and comparison. These structures support students in identifying essential information and organizing their thinking through problem schemas. Familiarity with such structures enables students to generalize strategies when encountering new problems. Therefore, teachers need to understand these logical structures and present a variety of problem types, recognizing that combining quantities does not always imply addition and separating quantities does not always imply subtraction. Through this approach, students gradually develop additive reasoning by understanding relationships between quantities rather than relying solely on computational procedures.

Understanding part–whole relationships is a significant conceptual achievement that allows students to decompose numbers and apply this understanding across various contexts (Björklund et al., 2020). This understanding includes the commutative property and the inverse relationship between addition and subtraction. As students develop fluency with addition facts, they learn that quantities can be composed of different parts without changing their total value. Story problems play an important role in supporting this understanding by presenting mathematical ideas within meaningful narratives that require students to read, interpret, and analyze situations involving addition and subtraction (Carpenter & Moser, 2020). In this context, teachers need to distinguish between semantic and computational forms of equations, as semantic representations align more closely with the structure of story problems and are easier for students to comprehend (Chiang & Chen, 2019). Therefore, this study aims to describe and analyze students' thinking processes in solving contextual addition and subtraction problems, focusing on patterns of representation and problem-solving strategies based on the structural framework proposed by Walle et al. (2016).

LITERATURE REVIEW

Revealing students' thought processes in solving addition and subtraction problems through the analysis of student work provides valuable insights for educators to refine instruction and enhance conceptual understanding in elementary mathematics classrooms (Amini et al., 2019). Addition and subtraction are fundamental mathematical operations that form the basis of arithmetic competence. Students' ability to perform calculations depends not only on procedural fluency but also on their understanding of numbers and their relationships. Arithmetic learning follows characteristic developmental paths, often described as learning trajectories, through which students gradually build more sophisticated strategies and conceptual understanding (Kutaka et al., 2023). However, when addition and subtraction are introduced primarily through formal notation, such as "2 + 3 = …," many students fail to grasp the underlying meaning of these operations, which can lead to persistent difficulties in both addition and subtraction.

Arithmetic operations play a crucial role in solving mathematical problems and addressing situations encountered in everyday life (Agbata et al., 2024). Addition and subtraction are core quantitative operations that support students' understanding of how numbers function and how strategies can be flexibly applied. Computational skills in addition and subtraction help students make decisions about procedures and develop multiple problem-solving strategies. Research suggests that when instruction emphasizes conceptual understanding alongside procedural practice, students' understanding and skills develop simultaneously. This emphasis aligns with curriculum expectations, which require students to explain and perform addition and subtraction involving numbers up to 999 in everyday contexts and to recognize the relationship between the two operations (Baroody et al., 2016). These competencies indicate that students are expected not only to calculate accurately but also to understand the concepts underlying addition and subtraction using a variety of techniques.

As students' progress beyond basic counting strategies, they begin to rely on arithmetic principles that govern numerical relationships. Mathematical principles are defined as fundamental properties inherent in mathematical systems rather than constructs developed independently by learners (Liu et al., 2019). One instructional context that supports the application of these principles is word problems. Word problems are verbal descriptions of problem situations presented in educational settings, where solutions are obtained by applying appropriate mathematical operations to the given numerical information (Dresen et al., 2020). Such problems may involve direct numerical expressions or be presented orally and contextually, requiring students to interpret the situation before selecting a strategy. Through this process, students learn to distinguish between surface features of problems and the underlying mathematical structure.

Word problems, also referred to as mathematical story problems, are designed to situate mathematics within real-life contexts and encourage students to apply mathematical concepts meaningfully (Verschaffel et al., 2020). These problems stimulate problem-solving skills by requiring students to identify relevant information, formulate solution strategies, and apply mathematical reasoning to reach a solution. Consequently, word problems support the development of critical, logical, and analytical thinking. Because of their instructional value, word problems have long been recognized as a central component of mathematics education and are widely used in curricula and assessments to achieve learning objectives related to problem solving (Mahmood et al., 2020). Their prominence has also made them a key focus of research in mathematics education.

The present study is grounded in the framework proposed by Van de Walle, particularly Chapter 9, which focuses on developing meaning for addition and subtraction operations (Walle et al., 2016). This framework categorizes addition and subtraction problems into four structural types: joining, separating, part–whole, and comparison. Understanding these structures helps students generalize strategies across similar problems and supports deeper conceptual understanding. Accordingly, teachers need to recognize these logical structures and design learning experiences that expose students to a variety of problem types, while also identifying which structures pose greater challenges for learners.

Each structure is characterized by distinct quantitative relationships (Walle et al., 2016). Joining problems involve an initial quantity, a change quantity, and a resulting total, with one of these quantities unknown. Separating problems also involve three quantities, but the initial amount represents the whole, distinguishing them from joining problems. Part–whole problems focus on the relationship between a whole and its constituent parts, emphasizing how two parts can be mentally or physically combined to form a whole. Comparison problems involve analyzing the difference between two quantities, often expressed in terms of “more than” or “less than,” and may require identifying the unknown difference or one of the compared quantities.

In addition to problem structures, Van de Walle emphasizes the importance of distinguishing between semantic and computational equations in addition and subtraction (Walle et al., 2016). Semantic equations reflect the sequence and meaning of events in a story problem, making them more accessible for students’ understanding and modeling of real situations. In contrast, computational equations are more abstract and are commonly used to efficiently obtain solutions by isolating the unknown quantity using appropriate operations. Understanding both forms enables students to connect contextual meaning with symbolic representation, thereby strengthening their overall problem-solving competence.

METHODOLOGY

Research Design

This study applies a qualitative research design using the case study method to explore how presents various variations of students' mathematical thinking skills based on how students answer mathematical problems. A case study enables researchers to examine a phenomenon within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident (Yin, 2018). This approach is appropriate when the research intends to understand complex social units involving multiple variables. The research design begins with identifying a problem or gap in understanding, which is followed by defining the purpose and formulating research questions. These research questions guide the selection of participants, design of instruments, and procedures for collecting data. In this study, the research questions are focused on how students demonstrate religious character in mathematics learning activities. To address this, the study uses three data collection techniques: interviews, classroom observations, and math problem-solving tests. The math tests contain story problems about addition and subtraction to assess students' reasoning in moral contexts. Classroom observations help reveal students’ religious behaviors, such as discipline, respect, and honesty during learning. The interviews aim to explore students’ thoughts, values, and motivations regarding their actions. Triangulation of these data sources improves the validity of findings. This multi-method strategy allows the researcher to examine the phenomenon from different angles.

Instruments Data Collection

The instrument form is a paper containing mathematical problems that are tested on students to see the pattern of answers to students' mathematical thinking abilities in completing the topic of addition and subtraction of numbers. The instruments used in this study consisted of mathematics problem-solving tests. The problem-solving tests contained story problems on addition and subtraction designed to reveal variations in students' mathematical thinking based on the strategies they used when answering the questions. The instruments were reviewed by one expert in mathematics education and differences in evaluation were resolved through discussion until consensus was reached, ensuring the clarity, relevance, and accuracy of each instrument.

This study employed a qualitative case study design to investigate the demonstrated by Grade V students during mathematics learning activities. The case consisted of five students from a public elementary school in Yogyakarta, identified as R01 to R05. These participants were selected using simple random sampling to ensure that each student in the class had an equal chance of being included. The case was clearly bounded by the research setting (one elementary school), the research period, and the specific focus of observation, allowing the data collection to be carried out intensively within a defined context. Using a case study design enabled the researcher to document real classroom situations in detail and capture the students' observable behaviors related to religious character.

Data Analysis

Data analysis in this case study follows an inductive process involving description, interpretation, conclusion, and significance. The first stage is descriptive, where the researcher organizes and familiarizes themselves with the raw data, identifying the who, what, when, and where of the case (Sugiyono, 2018). Next, the data is coded using open coding to allow themes and categories to emerge naturally. Open coding enables researchers to identify recurring patterns or meanings in participants' words and actions without forcing data into pre-existing frameworks (Bogdan & Biklen, 2007). Once codes are established, they are grouped into broader themes that represent the core of the findings. In this study, codes were related to aspects of religious character such as discipline, honesty, empathy, and responsibility. The analysis process involves not only looking at individual cases but also comparing them to see patterns across all five students. Constant comparative methods were used to refine categories and check their consistency. Visual matrices and narrative summaries helped organize the emerging themes. These techniques helped ensure a thorough and systematic approach to interpreting qualitative data.

To maintain the quality of the research process, several trustworthiness strategies were systematically applied. Credibility was ensured through methodological triangulation, member checking, and continuous observation of participants throughout the data collection period. Three validators were involved in reviewing the interview guidelines, observation sheets, and coding results. These validators consisted of two experts in qualitative research and one expert in character education. Differences in opinion among validators were resolved through sequential discussions until all reviewers agreed on the clarity, relevance, and consistency of each item. Dependability was strengthened by maintaining a detailed audit trail that documented every step of data collection and analysis. Confirmability was supported by keeping reflexive memos to minimize potential researcher bias and ensure that interpretations were grounded in the data. Transferability was enhanced by providing thick descriptions of the research context so that readers can determine the relevance of the findings to other settings (Miles & Huberman, 1994). These procedures collectively ensured that the data and analysis process met accepted standards of qualitative rigor.

RESULTS

This study presents various variations of students' mathematical thinking skills based on how students answer mathematical problems. In this study, testing was conducted by giving mathematical problems to 5 respondents in an elementary school in Yogyakarta. The respondents can be referred to as R01, R02, R03, R04, and R05. Meanwhile, the test items can be referred to as S01, S02, and S03. In S01, by testing 5 students with the topic of addition and subtraction of numbers, several variations of answers were obtained from R01, R02, R03, R04, and R05 which will be explained through pictures.

In addition, this study describes how teachers teach mathematics. The semantic and computational similarities identified in students' mathematical responses. Table 1 summarizes how students' written answers demonstrate patterns of meaning (semantic similarity) as well as similarities in the procedures or steps used to solve the problems (computational similarity). By comparing these two dimensions, the data provides a clearer picture of how each student approaches mathematical tasks, allowing the researcher to identify consistent reasoning strategies or notable variations across cases.

Table 1. Semantic and Computational Similarities

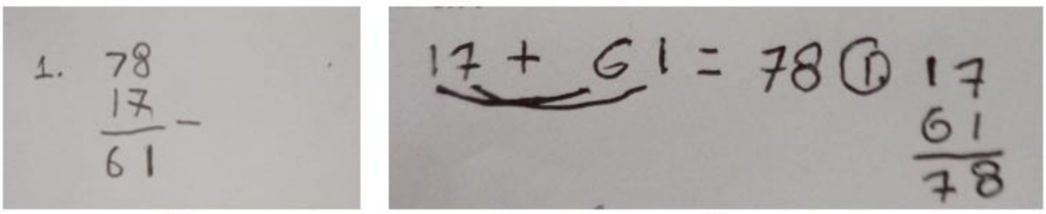
Information	Merger Issue	The Question of Separation
Results	9 + 5 = ?	14 – 5 = ?
Change	9 + ? = 14 (semantics) 14 − 9 = ? (computation)	14 - ? = 9 (semantics) 14 – 9 = ? (computation)
Initial Amount	? + 5 = 14 (semantics) 14 − 5 = ? (computation)	? - 5 = 9 (semantics) 14 – 9 = ? (computation)

The type of problem used in this study is to test the mathematical problems given with a written test. The mathematical problems are related to the most basic arithmetic topics, namely addition and subtraction operations. The problem items and instrument images can be explained as follows.

I am a number. If I add 17, I become 78. What is I? If you buy 167 orange candies and 89 chocolate candies. What is the total number of candies? And initially had 297 marbles. Andi gave some of the marbles to Rino. Now Andi only has 89 marbles left. How many did he give to his younger brother?

The mathematical problems are problems related to the topic of addition and subtraction. Van de Walle *et al.*, (2016) explains that there are four structures for presenting problems in adding and subtracting numbers. In the first problem, it is a type of *part problem. whole* refers to the concept of dividing a whole into interrelated parts that allow us to understand how the parts contribute to the whole. The second problem is a type of combination problem where the outcome is unknown. The third problem is a type of separation problem where the initial sum is the whole or the largest sum and is a separation where the change is unknown.

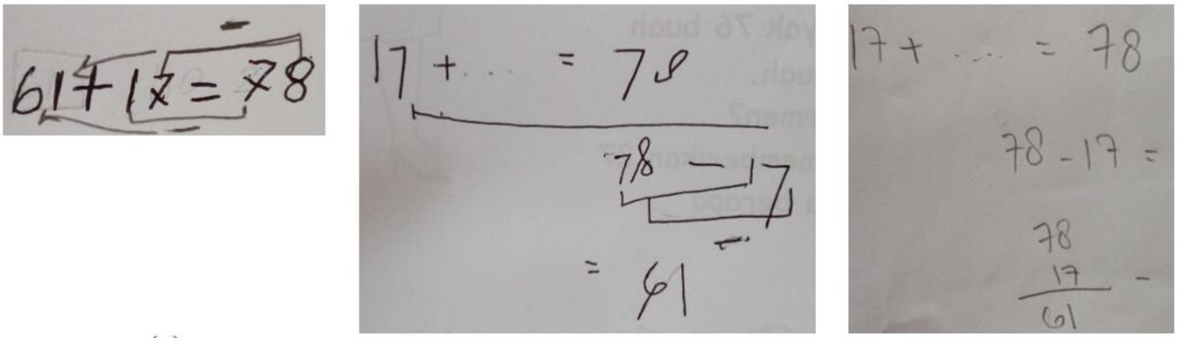
In this study, several students experienced difficulty adding numbers using existing techniques, namely the carrying technique for addition and the borrowing technique for subtraction. This problem occurred because students forgot to carry or borrow numbers. Even though the techniques were performed correctly, the answers remained incorrect. Therefore, researchers developed several addition and subtraction techniques without carrying and borrowing.



(a) (b)

Figure 1. R01's and R02's Response to S01

S01 describes a math problem with the topic of adding numbers presented in the form of a story problem. Based on Figure 1 (a), it is explained that R01 solves problems about addition operations using the stacking technique. R01 uses a commonly used technique. In the problem, there is a mathematical sentence of addition, namely 17 + ? = 78. R01's answer shows that R01 immediately answered by going directly to the calculation result and without writing the steps. R01 answered the problem by changing it into a calculation form to make it easier to answer the given math problem. In this case, R01 shows the ability to analyze the problem well so that when reading the story problem, he does not write it in the form of a mathematical sentence, but immediately changes it into a computational calculation to get a direct answer. Meanwhile, Figure 1 (b) shows that R02 also solved a subtraction problem using the stacking technique, similar to R01. However, R02 solved it by first using the semantic mathematical sentence and then calculating it by stacking it. R02 first writes the mathematical sentence in the form of a semantic sentence and then performs the operation directly using the place value method and then performs the operation again using the stacking method.



(a) (b) (c)

Figure 2. R03's, R04's and R05's Response to S01

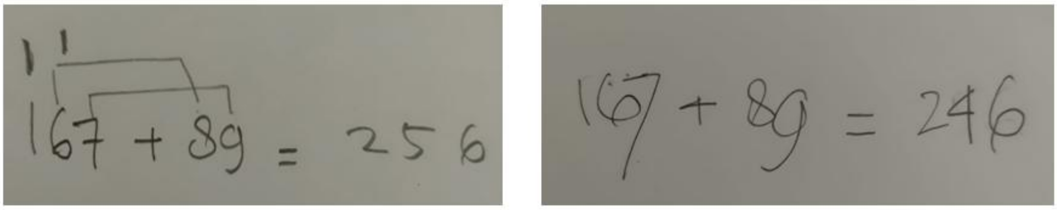
Furthermore, Figure 2 (a) shows that respondent R03 solved the addition problem using a place-value strategy by adding units to units and tens to tens. R03 first represented the story problem by writing a mathematical sentence to support observation and calculation. However, rather than solving the equation step by step in a formal manner, R03 directly determined the required value based on the written representation. Figure 2 (b) illustrates R04's strategy in solving problem S01. R04 initially wrote the equation 17 + ? = 78 as a representation of the story problem and retained this form during the calculation process. To find the unknown value, R04 applied a comparative subtraction strategy by subtracting the smaller number from the larger one, using a place-value approach similar to that employed by R03. In contrast, Figure 2 (c) depicts R05's approach, which involved first writing the semantic equation 17 + ? = 78 and then transforming it into a computational form, 78 − 17 = ?. This conversion enabled R05 to apply a systematic, step-by-step procedure to solve the problem. In performing the calculation, R05 used the downward stacking technique, consistent with the strategies observed in R01 and R02.

Based on the analysis of the five respondents, problem S01 was solved using a variety of strategies, although most students relied on the downward stacking method and converted semantic equations into computational forms to simplify calculations. This method involves subtracting digits sequentially according to their place value until the entire number has been processed. Through this approach, students demonstrate an understanding of numerical relationships and the importance of place value in subtraction. When a digit in the number being reduced is smaller than the corresponding digit in the number used for subtraction, students apply the borrowing process, which requires a conceptual understanding of regrouping. Pattern analysis within the stacking method helps students recognize how digits interact across place values and how subtraction results are generated step by step. These recurring solution patterns, as summarized in Table 2, indicate that procedural fluency is closely linked to students' understanding of number structure.

Table 2. Mapping of Solution Techniques in Answering Mathematics Problems S01

Respondents	Completion	Solution Techniques
R01	Changing to computational structure	Down stacking technique
R02	Changing to computational structure	Down stacking technique
R03	Using semantic structures	Sideways addition
R04	Using semantic structures	Sideways addition
R05	Changing to computational structure	Down stacking technique

Based on the interview results with the five respondents, it was found that students were rarely exposed to a variety of contextual mathematical problems. As a result, they initially experienced difficulty in translating story problems into appropriate addition or subtraction operations. Students often struggled to identify whether a given problem required an addition or a subtraction process. This difficulty stems from an insufficient understanding of the conceptual meaning of addition and subtraction. Without a clear grasp of the situations in which each operation is applied, students tend to misinterpret the context of story problems and select inappropriate operations. Therefore, students need to attend carefully to contextual cues and key information in the problem that indicate the required operation. With a solid understanding of the underlying meanings and situational contexts of addition and subtraction, students will be better equipped to select and apply the correct operation when solving story problems.

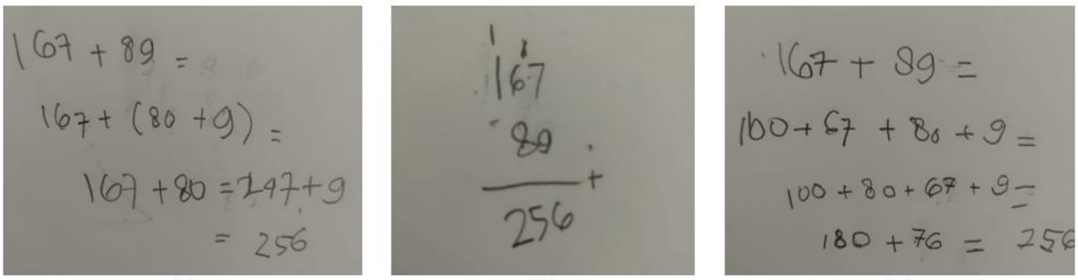


(a) (b)

Figure 3. R01's and R02's Response to S02

Figure 3(a) illustrates that respondent R01 solved question S02 using the sideways stacking technique, in which addition begins with the rightmost digit and proceeds sequentially to the left. When the sum of two digits exceeds nine, the extra value is carried over and added to the next place value. In this case, R01 first added the unit digits, 7 and 9, obtaining 16. The digit 6 was written as the unit result, while the carried value of 1 was stored. Next, R01 added the tens digits along with the carried value, resulting in 1 + 6 + 8 = 15. The digit 5 was recorded, and the remaining carried value of 1 was then added to the hundreds digit, yielding 1 + 1 = 2. Through this systematic process, R01 correctly obtained the final answer of 256 for question S02.

Figure 3(b) shows respondent R02 solving question S02 using a direct addition approach. Similar to R01, R02 began by adding the unit digits, 7 and 9, and obtained 16, recording only the digit 6 and carrying over the value of 1. However, when adding the tens digits, 6 and 8, R02 overlooked the previously carried value and calculated only 6 + 8 = 14. The digit 4 was written as the tens result, and the carried value of 1 from this step was then added to the hundreds digit, resulting in 1 + 1 = 2. Due to the omission of the carried value from the unit addition during the tens calculation, R02 produced an incorrect final answer of 246.



(a) (b) (c)

Figure 4. R03's, R04's and R05's response to S02

Based on Figure 4 (a), respondent R03 solved the S02 problem, using a number-partitioning strategy, in which numbers are decomposed into components that are easier to combine. R03 divided 89 into 80 and 9, then added these values sequentially to 167. First, 167 + 80 resulted in 247, which was then added to 9 to obtain the final result of 256. R03 successfully answered the question correctly. Interview results indicated that R03 found this strategy effective because it allowed the numbers to be arranged side by side in a flexible manner, making the addition process easier to manage and understand.

Figure 4 (b) shows that respondent R04 solved the same problem using the vertical stacking method. In this approach, digits are aligned according to place value, with units aligned with units, tens with tens, and the hundreds digit appearing only in 167, as 89 has no hundreds digit. R04 began by adding the units (7 + 9 = 16), writing 6 and carrying over 1. The carried value was then added to the tens digits (6 + 8 + 1 = 15), resulting in 5 with a carry of 1. Finally, the carried value was added to the hundreds digit (1 + 1 = 2), producing the correct answer of 256. According to the interview, R04 preferred this method because it provides a clear and structured way to organize numbers vertically, making it easier to add digits with the same place value systematically.

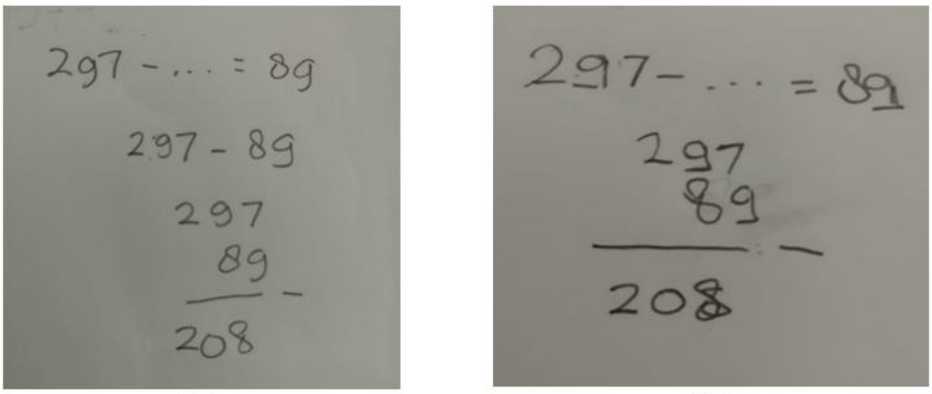
Based on Figure 4 (c), respondent R05 used a strategy similar to R03 but with a more explicit decomposition of both numbers. R05 broke down 167 into 100 and 67, and 89 into 80 and 9. The addition was performed by pairing compatible components, resulting in 100 + 80 = 180 and 67 + 9 = 76. These partial sums were then combined to obtain 180 + 76 = 256. R05 successfully solved the problem using this approach. Interview findings revealed that R05 referred to this approach as the "even number" strategy, which involved identifying round or familiar numbers (such as 80 or 100) first and then adding the remaining values. This strategy helped simplify the addition process by reducing computational complexity and breaking the task into smaller, more manageable steps, thereby increasing confidence and efficiency.

Furthermore, through the use of the even number strategy, R05 implicitly demonstrated an understanding of the commutative property of addition. R05 recognized that changing the order of addends does not affect the final result, which allowed flexibility in selecting and rearranging numbers during the calculation process. Repeated use of this strategy strengthened R05's conceptual understanding of addition and supported a more systematic and confident approach to problem solving. A summary of the characteristics of the strategies used by the five respondents is presented in Table 3.

Table 3. Mapping of Solution Techniques in Answering Mathematics Problems S02

Respondents	Completion	Solution Techniques
R01	Changing to computational structure	Sideways addition
R02	Changing to computational structure	Sideways addition
R03	Changing to computational structure	Sideways addition
R04	Changing to computational structure	Down stacking technique
R05	Changing to computational structure	Sideways addition

Furthermore, the problem of S03 focuses on subtraction in the context of a word problem, where the operation is indicated by the keyword "give," signaling that the initial quantity is reduced. Students' responses to this subtraction problem varied considerably, as illustrated in Figure 5 and Figure 6. These variations reflect differences in how students interpret contextual cues and translate them into mathematical representations. Overall, a clear understanding of subtraction concepts within word problems provides students with a strong foundation for developing problem-solving skills and for connecting mathematical operations to real-life situations.



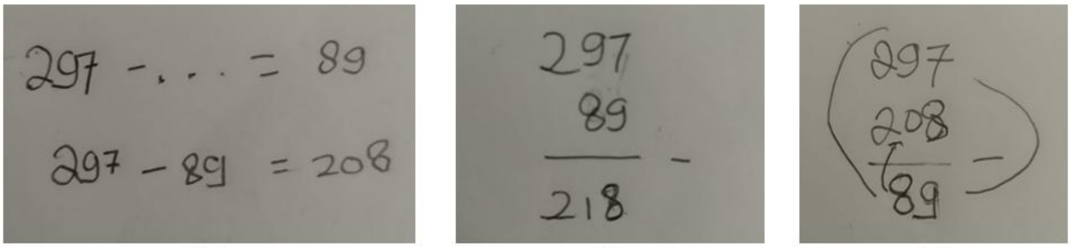
(a) (b)

Figure 5. R01's and R02's Response to S03

As shown in Figure 5 (a), respondent R01 began by representing the word problem through a number sentence, 297 − … = 89. R01 then transformed this semantic representation into a computational form by rearranging it as 297 − 89 = …, making it easier to perform the calculation.

Computational representations use mathematical symbols and notation to express relationships and operations clearly and systematically, and their effective use reflects an understanding of the underlying semantic structure of the word problem. After establishing the computational sentence, R01 applied the column subtraction method by subtracting digits according to their place values. This process involved borrowing when the digit in the number being reduced was smaller than the corresponding digit in the number being subtracted, ensuring accurate calculation across all place values. Through this structured approach, R01 successfully obtained the correct result.

Figure 5 (b) shows that respondent R02 employed a similar strategy by directly applying the vertical subtraction technique. R02 wrote the calculation in a stacked format, aligning digits by place value and subtracting systematically. This method is familiar to many students and is commonly used to minimize errors in subtraction. R02's response indicates a clear understanding of the problem context, enabling a smooth transition from interpretation to calculation. Similarly, respondent R03 demonstrated a rapid shift from understanding the word problem to performing the subtraction. This suggests that the concepts of addition and subtraction have been well internalized, allowing R03 to apply standard procedures efficiently and confidently.



(a) (b) (c)

Figure 6. R03's, R04's and R05's response to S03

Based on Figure 6 (a), respondent R03 presented the solution to problem S03 by first drawing a schematic representation to map the subtraction operation implied in the word problem. R03 rewrote the key quantities from the problem, identifying the initial number of marbles as 297 and the final number as 89. After clarifying this structure, R03 formulated the subtraction operation $297 - 89 = \ldots$ and proceeded to perform the calculation using a sideways stacking method. This approach involved subtracting digits according to their place values and is conceptually similar to the vertical stacking method, as both rely on place-value alignment and borrowing to obtain accurate results.

Figure 6 (b) shows that respondent R04 was unable to solve problem S03 correctly due to difficulties in interpreting the word problem. R04 wrote an incorrect mathematical sentence by representing the situation as an addition rather than a subtraction, which led to an incorrect final result. Although R04 attempted to apply the borrowing procedure during the calculation, a lack of understanding of when and how borrowing should be applied resulted in further errors. This finding indicates that misunderstanding the conceptual meaning of subtraction and borrowing can hinder students' ability to perform accurate calculations. Difficulties in recognizing the need to borrow from a higher place value are common sources of error in subtraction tasks.

Meanwhile, Figure 6 (c) indicates that respondent R05 solved problem S03 using a structure similar to that employed by R01 but relied directly on a semantic representation rather than converting the problem into a computational form. R05 wrote the semantic sentence based on the meaning of the word problem and used it to obtain the correct answer. This approach suggests a more mature level of mathematical thinking, as the student was able to operate directly from the contextual meaning without relying on formal symbolic transformation. Interview data from R01, R02, and R05 revealed that these students commonly used layered subtraction strategies, particularly for problems involving larger numbers or multiple place values. They reported that this method was their primary strategy and that they were not familiar with alternative approaches. The interviews also indicated that word problems were rarely used in their learning experiences, leading to epistemological barriers related to limited exposure to contextual problem-solving. A summary of these findings is presented in Table 4.

Table 4. Mapping of Solution Techniques in Answering Mathematics Problems S03

Respondents	Completion	Solution Techniques
R01	Changing to computational structure	Down stacking technique
R02	Changing to computational structure	Down stacking technique
R03	Using semantic structures	Sideways addition
R04	-	-
R05	Changing to semantic structure	Down stacking technique

DISCUSSION

Each student has a unique method or technique for solving mathematical addition problems (Fuson, 2020). Some students prefer the horizontal stacking method, where they align two numbers side by side and add them from the rightmost to the leftmost digit (Carpenter et al., 2015). Others choose the column addition technique, arranging numbers vertically and summing each digit place starting from the units (Reys et al., 2012). Some students also use mental addition strategies, performing calculations directly in their minds without writing them down (Dowker, 2005). Another group may apply compensation strategies, breaking one number into parts that are easier to add and then summing the partial results (Thompson & Saldanha, 2003). Visual learners often use pictorial or manipulative techniques by representing quantities with objects or images (Clements & Sarama, 2009). This variety shows that students develop personal preferences based on prior success or comfort levels. It is therefore essential for teachers to recognize the diverse strategies students bring to problem solving (Boaler, 2015). Offering multiple approaches allows students to choose methods that match their thinking styles and developmental stages (Walle et al., 2014).

Students' understanding of mathematical laws is often reflected in the techniques they use to perform addition (Rompas et al., 2023). Those who grasp the commutative property realize that changing the order of addends does not alter the sum (NCTM, 2000). For example, they know that $7 + 5$ yields the same result as $5 + 7$. Similarly, students familiar with the associative property understand that grouping numbers differently still results in the same total, such as $(3 + 4) + 2 = 3 + (4 + 2)$ (Walle et al., 2014). Others may apply the distributive property, particularly when addition involves multiplication, by distributing a value across addends before combining them (Haylock & Cockburn, 2017). The identity property is also key, where students learn that adding zero does not change the value of a number (Reys et al., 2012). In contrast, the inverse property of addition helps students solve for missing values by using subtraction (Charles, 1987). These mathematical laws provide a foundation for flexible thinking and efficiency in computation (Baroody, 2006). Students who internalize these principles are better able to select appropriate strategies and explain their reasoning.

Addition and subtraction with whole numbers are basic arithmetic skills that must be mastered by elementary school students from an early age (Siegler & Ramani, 2009). However, some students experience difficulties when using regrouping or borrowing techniques in subtraction. At this developmental stage, many children still think concretely and are not yet ready to engage with abstract symbols and logic (Piaget, 1972). Although they may understand definitions and procedures, they struggle to generalize or create their own concepts. Therefore, mathematics instruction at the elementary level must be adapted to students' cognitive characteristics. Activities should relate to students' real-world experiences to enhance relevance and engagement (Nunes & Bryant, 1996). In practice, many students add up all the numbers they see in a story problem without understanding the context. Misunderstanding of place value also appears frequently in student responses (Ross, 1989). This suggests the need for more focused and meaningful instruction on number sense and base-ten concepts.

Based on the previous explanations, students face three major obstacles in learning addition and subtraction: epistemological, didactic, and cognitive. First, students lack experience in solving contextual problems due to the scarcity of real-life application in daily learning (Zulkardi, 2002). Second, didactic obstacles arise from teaching methods that focus only on explanation without providing space for exploration and construction of knowledge (Gravemeijer & Doorman, 1999). Third, cognitive barriers are evident when students are expected to operate abstractly without sufficient concrete grounding (Baroody, 2006). Teachers need to adopt active learning approaches to develop students' logical and structured thinking. One effective strategy is the use of visual aids and interrogative language to support students' interpretation of story problems (Clements & Sarama, 2009). Incorporating images into problem contexts can enhance comprehension and reduce cognitive load. By designing tasks that integrate meaning, strategy, and representation, teachers can support deeper understanding. Ultimately, a student-centered, inquiry-based approach is essential for building long-term mathematical thinking.

CONCLUSION

This study confirms that elementary students demonstrate a variety of strategies when solving addition and subtraction problems, including vertical stacking, horizontal arrangement, and transforming semantic information into computational expressions, reflecting their understanding of mathematical principles such as the commutative and associative properties. Some students successfully applied mental strategies and estimation, while others relied heavily on procedural methods, and those with stronger comprehension of story contexts produced more accurate mathematical sentences. However, difficulties in subtraction involving borrowing and challenges in interpreting story problems indicate gaps between conceptual and procedural understanding, underscoring the need for improved instructional support. This study is limited by its focus on a single school with only five participants, restricting the representativeness and generalizability of the findings, and the use of tasks limited to addition and subtraction further narrows the scope of interpretation. Despite these limitations, the findings hold important theoretical and practical implications: theoretically, the study contributes to understanding how semantic and computational processes interact in shaping children's arithmetic reasoning; practically, the results highlight the need for instructional approaches that help students transition from contextual comprehension to formal mathematical representations, including scaffolding strategies, explicit exploration of multiple solution methods, and opportunities for students to articulate their reasoning to strengthen early mathematics learning.

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